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**CCGPS**

**Frameworks**

**Teacher Edition**

**Mathematics**



**Unit 2**

**Reasoning with Equations and Inequalities**

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# OVERVIEW

In this unit students will:

* solve linear equations in one variable.
* solve linear inequalities in one variable.
* solve a system of two equations in two variables by using multiplication and addition.
* solve a system of two equations in two variables graphically
* graph a linear inequality in two variables.
* graph a system of two linear inequalities in two variables.

By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. The second unit of Coordinate Algebra expands the previously learned concepts of solving and graphing linear equations and inequalities, focusing on the reasoning and understanding involved in justifying the solution. Students are asked to explain and justify the mathematics required to solve both simple equations and systems of equations in two variables using both graphing and algebraic methods. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. All of this work is grounded on understanding quantities and on relationships between them.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight process standards should be addressed constantly as well. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under “Evidence of Learning” be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

# STANDARDS ADDRESSED IN THIS UNIT

# KEY STANDARDS

**Understand solving equations as a process of reasoning and explain the reasoning**

**MCC9‐12.A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**Solve equations and inequalities in one variable**

**MCC9‐12.A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

**Solve systems of equations**

**MCC9‐12.A.REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

**MCC9‐12.A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**Represent and solve equations and inequalities graphically**

**MCC9‐12.A.REI.12** Graph the solutions to a linear inequality in two variables as a half‐plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half‐planes.

**Standards for Mathematical Practice**

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. **Make sense of problems and persevere in solving them.** High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
2. **Reason abstractly and quantitatively.** High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
3. **Construct viable arguments and critique the reasoning of others.** High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. **Model with mathematics.** High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
5. **Use appropriate tools strategically.** High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
6. **Attend to precision.** High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
7. **Look for and make use of structure.** By high school, students look closely to discern a pattern or structure. In the expression *x*2 + 9*x* + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(*x* – *y*)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.
8. **Look for and express regularity in repeated reasoning.** High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding (*x* – 1)(*x* + 1), (*x* – 1)(*x*2 + *x* + 1), and (*x* – 1)(*x*3 + *x*2 + *x* + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content**

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who do not have an understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a missing mathematical knowledge effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

# ENDURING UNDERSTANDINGS

* Solve linear equations and inequalities is one variable.
* Graph linear equations and inequalities in two variables.
* Solve systems of linear equations in two variables exactly and approximately.
* Create linear equations and inequalities in one variable and use them in a contextual situation to solve problems.
* Create equations in two or more variables to represent relationships between quantities.
* Graph equations in two variables on a coordinate plane and label the axes and scales.
* Write and use a system of equations and/or inequalities to solve real world problems.

# ESSENTIAL QUESTIONS

* How do I justify the solution to an equation?
* How do I solve an equation in one variable?
* How do I solve an inequality in one variable?
* How do I prove that a system of two equations in two variables can be solved by multiplying and adding to produce a system with the same solutions?
* How do I solve a system of linear equations graphically?
* How do I graph a linear inequality in two variables?
* How do I graph a system of linear inequalities in two variables?

# CONCEPTS/SKILLS TO MAINTAIN

Students may not realize the importance of unit conversion in conjunction with computation when solving problems involving measurement. Since today’s calculating devices often display 8 to 10 decimal places, students frequently express answers to a much greater degree of precision than is required.

Measuring commonly used objects and choosing proper units for measurement are part of the mathematics curriculum prior to high school. In high school, students experience a broader variety of units through real-world situations and modeling, along with the exploration of the different levels of accuracy and precision of the answers.

An introduction to the use of variable expressions and their meaning, as well as the use of variables and expressions in real-life situations, is included in the Expressions and Equations Domain of Grade 7.

Working with expressions and equations, including formulas, is an integral part of the curriculum in Grades 7 and 8. In high school, students explore in more depth the use of equations and inequalities to model real-world problems, including restricting domains and ranges to fit the problem’s context, as well as rewriting formulas for a variable of interest.

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess to determine whether instructional time should be spent on conceptual activities that help students develop a deeper understanding of these ideas.

* Using the Pythagorean Theorem
* Understanding slope as a rate of change of one quantity in relation to another quantity
* Interpreting a graph
* Creating a table of values
* Working with functions
* Writing a linear equation
* Using inverse operations to isolate variables and solve equations
* Maintaining order of operations
* Understanding notation for inequalities
* Being able to read and write inequality symbols
* Graphing equations and inequalities on the coordinate plane
* Understanding and use properties of exponents
* Graphing points
* Choosing appropriate scales and label a graph

# SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

**The definitions below are for teacher reference only and are not to be memorized by the students.** Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school children. **Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

<http://www.amathsdictionaryforkids.com/>

This web site has activities to help students more fully understand and retain new vocabulary.

<http://intermath.coe.uga.edu/dictnary/homepg.asp>

Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

* **Algebra:** The branch of mathematics that deals with relationships between numbers, utilizing letters and other symbols to represent specific sets of numbers, or to describe a pattern of relationships between numbers.
* **Coefficient:** A number multiplied by a variable.
* **Equation:** A number sentence that contains an equals symbol.
* **Expression:** A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.
* **Inequality**: Any mathematical sentence that contains the symbols > (greater than), < (less than), < (less than or equal to), or > (greater than or equal to).
* **Ordered Pair**:  A pair of numbers, (x, y), that indicate the position of a point on a Cartesian plane.
* **Substitution:** To replace one element of a mathematical equation or expression with another.
* **Variable:** A letter or symbol used to represent a number.

**The Properties of Operations**

Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

|  |  |
| --- | --- |
| *Associative property of addition* | (*a + b*) *+ c = a +* (*b + c*) |
| *Commutative property of addition* | *a + b = b + a* |
| *Additive identity property of 0* | *a +* 0 *=* 0 + *a* = *a* |
| *Existence of additive inverses* | For every *a* there exists –*a* so that *a* + (–*a*) = (–*a*) + *a* = 0. |
| *Associative property of multiplication* | (*a* × *b*) × *c = a* × (*b* × *c*) |
| *Commutative property of multiplication* | *a* × *b = b* × *a* |
| *Multiplicative identity property of 1* | *a* × 1 *=* 1 × *a* = *a* |
| *Existence of multiplicative inverses* | For every *a* ≠ 0 there exists 1/*a* so that *a* × 1/*a* = 1/*a* × *a* = 1. |
| *Distributive property of multiplication over addition* | *a* × (*b* + *c*) *= a* × *b* + *a* × *c* |

**The Properties of Equality**

Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.

|  |  |
| --- | --- |
| *Reflexive property of equality* | *a* = *a* |
| *Symmetric property of equality* | If *a = b*, then *b = a.* |
| *Transitive property of equality* | If *a = b* and *b = c*, then *a = c.* |
| *Addition property of equality* | If *a = b*, then *a + c = b + c.* |
| *Subtraction property of equality* | If *a = b*, then *a* – *c* = *b* – *c.* |
| *Multiplication property of equality* | If *a = b*, then *a* × *c* = *b* × *c.* |
| *Division property of equality* | If *a = b* and *c* ≠ 0, then *a* ÷ *c* = *b* ÷ *c.* |
| *Substitution property of equality* | If *a* = *b*, then *b* may be substituted for *a* in any expression containing *a*. |

# CLASSROOM ROUTINES

The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities as estimating, analyzing data, describing patterns, and answering daily questions. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, how to access classroom technology such as computers and calculators. An additional routine is to allow plenty of time for students to explore new materials before attempting any directed activity with these new materials.  The regular use of routines is important to the development of students' number sense, flexibility, fluency, collaborative skills and communication. These routines contribute to a rich, hands-on standards-based classroom and will support students’ performances on the tasks in this unit and throughout the school year.

# STRATEGIES FOR TEACHING AND LEARNING

* Students should be actively engaged by developing their own understanding.
* Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols and words.
* Interdisciplinary and cross-curricular strategies should be used to reinforce and extend the learning activities.
* Appropriate manipulatives and technology should be used to enhance student learning.
* Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
* Students should write about the mathematical ideas and concepts they are learning.
* Consideration of all students should be made during the planning and instruction of this unit. Teachers need to consider the following:
  + What level of support do my struggling students need in order to be successful with this unit?
  + In what way can I deepen the understanding of those students who are competent in this unit?
  + What real life connections can I make that will help my students utilize the skills practiced in this unit

# EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

* justify the solution of a linear equation and inequality in one variable.
* justify the solution to a system of 2 equations in two variables.
* solve a system of linear equations in 2 variables by graphing.
* graph a linear inequality in 2 variables.
* graph a system of linear inequalities in 2 variables.

# TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all Coordinate Algebra students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task).

|  |  |  |
| --- | --- | --- |
| **Task Name** | **Task Type**  ***Grouping Strategy*** | **Content Addressed** |
| Jaden’s Phone Plan | Scaffolding Task  *Individual/Partner Task* | Model and write an equation in one variable Represent constraints with inequalities |
| Solving Systems of Equations Algebraically  Part 1 and Part 2 | Scaffolding Task  *Individual/Partner Task* | Justify the solution to a system of equations by graphing and substituting values into the system  Create new equations by multiplying one or more equations by a constant  Solve systems of equations by elimination |
| Summer Job | Scaffolding Task  *Partner/Small Group Task* | Modeling linear patterns  Creating equation and inequalities in one and two variables to represent relationships |
| Graphing Inequalities | Scaffolding Task  *Partner/Small Group Task* | Modeling with inequalities |
| Family Outing | Culminating Task  *Partner/Small Group Task* | Graph equations on coordinate axes with labels and scales  Solve systems of equations  Solve systems of inequalities  Determining constraints |

## Jaden’s Phone Plan

**Mathematical Goals**

* Create one-variable linear equations and inequalities from contextual situations.
* Solve and interpret the solution to multi-step linear equations and inequalities in context.

**Common Core State Standards**

**MCC9‐12.A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**MCC9‐12.A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**Introduction**

In this task, students will solve a series of linear equations and inequality word problems to help Jaden choose a cell phone plan. In order to help Jaden, students must explain in detail each step of the problem and justify the answer.

**Materials**

* Paper
* Pencil

**Jaden’s Phone Plan**

Jaden has a prepaid phone plan (Plan *A*) that charges 15 cents for each text sent and 10 cents per minute for calls.

***Comment:***

***As students solve equations throughout this task, have them explain each step using properties of operations or properties of equality.***

1. If Jaden uses only text, write an equation for the cost *C* of sending *t* texts.

***C = $.15t***

* 1. How much will it cost Jaden to send 15 texts? Justify your answer.

***C = $.15 \* 15***

***C = $ 2.25***

* 1. If Jaden has $6, how many texts can he send? Justify your answer.

***C = $.15t***

***$6 = $.15t***

***$6/$.15 = t***

***t = 40 texts***

1. If Jaden only uses the talking features of his plan, write an equation for the cost *C* of talking *m* minutes.

***C = $.10m***

* 1. How much will it cost Jaden to talk for 15 minutes? Justify your answer.

***C = $.10 \* 15***

***C = $1.50***

* 1. If Jaden has $6, how many minutes can he talk? Justify your answer.

***C= $.10m***

***$6 = .10m***

***$6/.10 = m***

***m = 60 minutes***

1. If Jaden uses both talk and text, write an equation for the cost *C* of sending t texts and talking *m* minutes.

***C = $.15t + $.10m***

* 1. How much will it cost Jaden to send 7 texts and talk for 12 minutes? Justify your answer.

***C = $.15t + $.10m***

***C = $.15\*7 + $.10\*12***

***C = $1.05 + $1.20***

***C = $2.25***

* 1. If Jaden wants to send 21 texts and only has $6, how many minutes can he talk? Will this use all of his money? If not, will how much money will he have left? Justify your answer.

***C = $.15t + $.10m***

***$6 = $.15\*21 + $.10m***

***$6 = $3.15 + .10m***

***$6 - $3.15 = .10m***

***$2.85 = .10m***

***$2.85/.10 = m***

***m = 28.5 minutes***

***Since most carriers will charge a full minute for any fraction of a minute, Jaden can talk for 28 minutes. He will have $0.05 left over if he talks for 28 minutes.***

Jaden discovers another prepaid phone plan (Plan *B*) that charges a flat fee of $15 per month, then $.05 per text sent or minute used.

1. Write an equation for the cost of Plan *B*.

***C = $15 + $.05n (Since the cost of text and talk are the same, the same variable can represent both.)***

In an average month, Jaden sends 200 texts and talks for 100 minutes.

1. Which plan will cost Jaden the least amount of money? Justify your answer.

***Plan A: C = $.15t + $.10m***

***C = $.15\*200 + $.10\*100***

***C = $30 + $10***

***C = $40***

***Plan B: C = 15 + .05n***

***C = $15 + $.05(200 + 100)***

***C = $15 + $.05(300)***

***C = $15 + $15***

***C = $30***

***Based on Jaden’s average usage, the cost for Plan A is $40 per month and the cost for Plan B is $30 per month. Therefore, Plan B will cost Jaden the least amount of money.***

## Jaden’s Phone Plan

**Mathematical Goals**

* Create one-variable linear equations and inequalities from contextual situations.
* Solve and interpret the solution to multi-step linear equations and inequalities in context.

**Common Core State Standards**

**MCC9‐12.A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**MCC9‐12.A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

Jaden has a prepaid phone plan (Plan *A*) that charges 15 cents for each text sent and 10 cents per minute for calls.

1. If Jaden uses only text, write an equation for the cost *C* of sending *t* texts.
   1. How much will it cost Jaden to send 15 texts? Justify your answer.
   2. If Jaden has $6, how many texts can he send? Justify your answer.
2. If Jaden only uses the talking features of his plan, write an equation for the cost *C* of talking *m* minutes.
   1. How much will it cost Jaden to talk for 15 minutes? Justify your answer.
   2. If Jaden has $6, how many minutes can he talk? Justify your answer.
3. If Jaden uses both talk and text, write an equation for the cost *C* of sending t texts and talking *m* minutes.
   1. How much will it cost Jaden to send 7 texts and talk for 12 minutes? Justify your answer.
   2. If Jaden wants to send 21 texts and only has $6, how many minutes can he talk? Will this use all of his money? If not, will how much money will he have left? Justify your answer.

Jaden discovers another prepaid phone plan (Plan *B*) that charges a flat fee of $15 per month, then $.05 per text sent or minute used.

1. Write an equation for the cost of Plan *B*.

In an average month, Jaden sends 200 texts and talks for 100 minutes.

1. Which plan will cost Jaden the least amount of money? Justify your answer.

## Solving System of Equations Algebraically

**Mathematical Goals**

* Model and write an equation in one variable and solve a problem in context.
* Create one-variable linear equations and inequalities from contextual situations.
* Represent constraints with inequalities.
* Solve word problems where quantities are given in different units that must be converted to understand the problem.

**Common Core State Standards**

**MCC9‐12.A.REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

**MCC9‐12.A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**Introduction**

In this task, students justify the solution to a system of equations by both graphing and substituting values into the system. Students will then show that multiplying one or both equations in a system of equations by a constant creates a new system with the same solutions as the original. This task will lead into using the elimination method for solving a system of equations algebraically.

**Materials**

* Ruler
* Calculator
* Notebook paper
* Pencil

***Comment:***

***As students solve equations throughout this task, have them continue to explain each step using properties of operations or properties of equality.***

Part 1:

You are given the following system of two equations: *x* + 2*y* = 16

3*x* – 4*y* = -2

1. What are some ways to prove that the ordered pair (6, 5) is a solution?

***Graphing and direct substitution are two methods for proving that (6,5) is a solution.***

* 1. Prove that (6, 5) is a solution to the system by graphing the system.



* 1. Prove that (6, 5) is a solution to the system by substituting in for both equations.

***x + 2y = 16 3x - 4y = –2***

***6 + 2\*5 = 16 3\*6 - 4\*5 = –2***

***6 + 10 = 16 18 – 20 = –2***

***16 = 16 –2 = –2***

***The solution (6, 5) works for both equations.***

1. Multiply both sides of the equation ***x* + 2*y* = 16** by the constant ‘7’. Show your work.

**7**\*(*x* + 2*y*) = **7**\*16

7\*x + 7 \* 2y = 112

***7x + 14y = 112*** New Equation

* 1. Does the new equation still have a solution of (6, 5)? Justify your answer.

***7x + 14y = 112***

***7\*6 + 14\*5 = 112***

***42 + 70 = 112***

* 1. Why do you think the solution to the equation never changed when you multiplied by the ‘7’?

***Answers may vary.***

1. Did it have to be a ‘7’ that we multiplied by in order for (6, 5) to be a solution?

***Answers may vary.***

* 1. Multiply ***x* + 2*y* = 16** by three other numbers and see if (6, 5) is still a solution.

Students may pick any constant to multiply the original equation by. As long as the multiplication is correct, the solution (6, 5) will still work.

* + 1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
    2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
    3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
  1. Did it have to be the first equation ***x* + 2*y* = 16** that we multiplied by the constant for (6, 5) to be a solution? Multiply **3*x* – 4*y* = -2** by ‘7’? Is (6, 5) still a solution?

***Use this exercise to help students discover that multiplying the equation by any constant will not change the solution.***

***7(3x – 4y) = 7\*-2***

***21x – 28y = –14***

***21\*6 – 28 \*5 = –14***

***126 – 140 = –14***

***–14 = –14***

* 1. Multiply **3*x* – 4*y* =** –**2** by three other number and see if (6, 5) is still a solution.
     1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
     2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
     3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Summarize your findings from this activity so far. Consider the following questions: What is the solution to a system of equations and how can you prove it is the solution?

Does the solution change when you multiply one of the equations by a constant?

Does the value of the constant you multiply by matter?

Does it matter which equation you multiply by the constant?

***Answers will vary, but the purpose of this task is for students to discover that multiplying an equation by a constant does not change the solution to the equation, leading into the elimination method for solving a system of equations.***

Let’s explore further with a new system. 5*x* + 6*y* = 9

4*x* + 3*y* = 0

1. Show by substituting in the values that (–3, 4) is the solution to the system.

***5x + 6y = 9 4x + 3y = 0***

***5(–3) + 6(4) = 9 4(–3) + 3(4) = 0***

***–15 + 24 = 9 –12 + 12 = 0***

***9 = 9 0 = 0***

1. Multiply **4*x* + 3*y* = 0** by ‘-5’. Then add your answer to **5*x* + 6*y* = 9**. Show your work below.

(–5)\*(4x + 3y) = (-5)\*0 🡪 ***\_\_\_–20x – 15y = 0\_\_\_*** Answer

+  ***5x + 6y = 9\_\_\_\_\_\_***

***\_\_\_–15x – 9y = 9\_\_\_\_*** New Equation

1. Is (–3, 4) still a solution to the new equation? Justify your answer.

***–15x – 9y = 9***

***–15(–3) – 9(4) = 9***

***45 – 36 = 9***

1. Now multiply 4*x* + 3*y* = 0 by ‘-2’. Then add your answer to **5*x* + 6*y* = 9**. Show your work below.

***–8x – 6y = 0***

***+ 5x + 6y = 9***

***–3x + 0y = 9***

***–3x = 9***

***x = 9/(–3)***

***x = –3***

* 1. What happened to the *y* variable in the new equation?

***It canceled out (became 0y), therefore being eliminated from the equation.***

* 1. Can you solve the new equation for *x*? What is the value of *x*? Does this answer agree with the original solution?

***See work above.***

***The original solution was (–3, 4), so a value of x = –3 does agree.***

* 1. How could you use the value of *x* to find the value of *y* from one of the original equations? Show your work below.

***Substitute the value of x into one of the equations to find the value of ‘y’.***

***5x + 6y = 9***

***5(3) + 6y = 9***

***–15 + 6y = 9***

***6y = 9 + 15***

***6y = 24***

***y = 24/6***

***y = 4***

The method you have just used is called the Elimination Method for solving a system of equations. When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Use the Elimination Method to solve the following system of equations:

1. –3*x* + 2*y* = –6 10. –5*x* + 7*y* = 11

5*x* – 2*y* = 18 5*x* + 3*y* = 19

***–3x + 2y = -6 –5x + 7y = 11***

***5x – 2y = 18 5x + 3y = 19***

***2x + 0y = 12 0x + 10y = 30***

***2x = 12 10y = 30***

***x = 6 y = 3***

***5(6) – 2y = 18 –5x + 7(3) = 11***

***30 – 2y = 18 –5x + 21 = 11***

***–2y = –12 –5x = –10***

***y = 6 x = 2***

***Solution :***

***(6, 6) (2, 3)***

***Check:***

***–3(6) + 2(6) = -6 –5(2) + 7(3) = 11***

***5(6) – 2(6) = 18 5(2) + 3(3) = 19***

Part 2:

When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Sometimes it is necessary to multiply one or both of the original equations by a constant. The equations are then added together and one of the variables is eliminated. Use the Elimination Method to solve the following system of equations:

1. 4*x* + 3*y* = 14 (Equation 1)

–2*x* + *y* = 8 (Equation 2)

Choose the variable you want to eliminate.

* 1. To make the choice, look at the coefficients of the *x* terms and the *y* terms. The coefficients of *x* are ‘4’ and ‘–2’. If you want to eliminate the *x* variable, you should multiply Equation 2 by what constant?

***Multiply the 2nd equation by the constant ‘2’.***

* + 1. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the *x* variable?

***2(–2x + y) = 2(8)***

***–4x + 2y = 16***

***–4x + 2y = 16 (New Equation 2)***

***+ 4x + 3y = 14 (Equation 1)***

***0x + 5y = 30***

***5y = 30***

***y = 6***

***The x variable was eliminated***.

* + 1. Solve the equation for *y*. What value did you get for *y*?

***See above.***

* + 1. Now substitute this value for *y* in Equation 1 and solve for *x*. What is your ordered pair solution for the system?

***4x + 3y = 14***

***4x + 3(6) = 14***

***4x +18 = 14***

***4x = –4***

***x = –1***

* + 1. Substitute your solution into Equation 1 and Equation 2 to verify that it is the solution for the system.

***Solution: (–1, 6)***

***4x + 3y = 14 4(–1) + 3(6) = 14***

***–2x + y = 8 –2(–1) + 6 = 8***

* 1. The coefficients of *y* are ‘3’ and ‘1’. If you want to eliminate the *y* term, you should multiply Equation 2 by what constant?

***Multiply Equation 2 by the constant (-3).***

* + 1. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the *y* variable?

***(–3)( –2x + y) = (–3)8***

***6x – 3y = –24 (New Equation 2)***

***6x – 3y = –24***

***+ 4x + 3y = 14***

***10x + 0y = –10***

***10x = –10***

***x = –1***

* + 1. Solve the equation for *x*. What value did you get for *x*?

***See above.***

* + 1. Now substitute this value for *x* in Equation 1 and solve for *y*. What is your ordered pair solution for the system?

***4x + 3y = 14***

***4(–1) + 3y = 14***

***3y = 18***

***y = 6***

***Solution: (–1, 6)***

Use your findings to answer the following in sentence form:

* 1. Is the ordered pair solution the same for either variable that is eliminated? Justify your answer.

***Answers may vary, but students should realize that the solution is the same for either variable that is eliminated.***

* 1. Would you need to eliminate both variables to solve the problem? Justify your answer.

***Answers may vary, but since either elimination yields the same answer, there is no need to eliminate both ways.***

* 1. What are some things you should consider when deciding which variable to eliminate? Is there a wrong variable to eliminate?

***Answers may vary, but there is no wrong variable to eliminate. Hopefully students have discovered that considering the coefficients of each variable will sometimes lessen the work involved in eliminating a particular variable.***

* 1. How do you decide what constant to multiply by in order to make the chosen variable eliminate?

***Answers may vary, but once the variable to be eliminated is chosen, the coefficients of that variable must be opposites so that the variable will eliminate.***

Use the elimination method to solve the following systems of equations. Verify your solution by substituting it into the original system.

1. 3*x* + 2*y* = 6 3. –6*x* + 5*y* = 4 4. 5*x* + 6*y* = –16

–6*x* – 3*y* = –6 7*x* – 10*y* = –8 2*x* + 10*y* = 5

***(–2, 6) (0, .8) (–95/19, 57/38)***

## Solving System of Equations Algebraically

**Mathematical Goals**

* Model and write an equation in one variable and solve a problem in context.
* Create one-variable linear equations and inequalities from contextual situations.
* Represent constraints with inequalities.
* Solve word problems where quantities are given in different units that must be converted to understand the problem.

**Common Core State Standards**

**MCC9‐12.A.REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

**MCC9‐12.A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

Part 1:

You are given the following system of two equations: *x* + 2*y* = 16

3*x* – 4*y* = –2

1. What are some ways to prove that the ordered pair (6, 5) is a solution?
   1. Prove that (6, 5) is a solution to the system by graphing the system.



* 1. Prove that (6, 5) is a solution to the system by substituting in for both equations.

1. Multiply both sides of the equation ***x* + 2*y* = 16** by the constant ‘7’. Show your work.

**7**\*(*x* + 2*y*) = **7**\*16

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ New Equation

* 1. Does the new equation still have a solution of (6, 5)? Justify your answer.
  2. Why do you think the solution to the equation never changed when you multiplied by the ‘7’?

1. Did it have to be a ‘7’ that we multiplied by in order for (6, 5) to be a solution?
   1. Multiply ***x* + 2*y* = 16** by three other numbers and see if (6, 5) is still a solution.
      1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
      2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
      3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
   2. Did it have to be the first equation ***x* + 2*y* = 16** that we multiplied by the constant for (6, 5) to be a solution? Multiply **3*x* – 4*y* =** –**2** by ‘7’? Is (6, 5) still a solution?
   3. Multiply **3*x* – 4*y* =** –**2** by three other number and see if (6, 5) is still a solution.
      1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
      2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
      3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Summarize your findings from this activity so far. Consider the following questions: What is the solution to a system of equations and how can you prove it is the solution?

Does the solution change when you multiply one of the equations by a constant?

Does the value of the constant you multiply by matter?

Does it matter which equation you multiply by the constant?

Let’s explore further with a new system. 5*x* + 6*y* = 9

4*x* + 3*y* = 0

1. Show by substituting in the values that (-3, 4) is the solution to the system.
2. Multiply **4*x* + 3*y* = 0** by ‘-5’. Then add your answer to **5*x* + 6*y* = 9**. Show your work below.

(–5)\*(4*x* + 3*y*) = (–5)\*0 🡪 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Answer

+ 5*x* + 6*y* = 9\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ New Equation

1. Is (–3, 4) still a solution to the new equation? Justify your answer.
2. Now multiply 4*x* + 3*y* = 0 by ‘–2’. Then add your answer to **5*x* + 6*y* = 9**. Show your work below.
   1. What happened to the *y* variable in the new equation?
   2. Can you solve the new equation for *x*? What is the value of *x*? Does this answer agree with the original solution?
   3. How could you use the value of *x* to find the value of *y* from one of the original equations? Show your work below.

The method you have just used is called the Elimination Method for solving a system of equations. When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Use the Elimination Method to solve the following system of equations:

1. –3*x* + 2*y* = -6 10. –5*x* + 7*y* = 11

5*x* – 2*y* = 18 5*x* + 3*y* = 19

**Solving System of Equations Algebraically**

Part 2:

When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Sometimes it is necessary to multiply one or both of the original equations by a constant. The equations are then added together and one of the variables is eliminated. Use the Elimination Method to solve the following system of equations:

1. 4*x* + 3*y* = 14 (Equation 1)

–2*x* + *y* = 8 (Equation 2)

Choose the variable you want to eliminate.

* 1. To make the choice, look at the coefficients of the *x* terms and the *y* terms. The coefficients of *x* are ‘4’ and ‘–2’. If you want to eliminate the *x* variable, you should multiply Equation 2 by what constant?
     1. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the *x* variable?
     2. Solve the equation for *y*. What value did you get for *y*?
     3. Now substitute this value for *y* in Equation 1 and solve for *x*. What is your ordered pair solution for the system?
     4. Substitute your solution into Equation 1 and Equation 2 to verify that it is the solution for the system.
  2. The coefficients of *y* are ‘3’ and ‘1’. If you want to eliminate the *y* term, you should multiply Equation 2 by what constant?
     1. Multiply Equation 2 by this constant. Then add your answer equation to Equation 1. What happened to the *y* variable?
     2. Solve the equation for *x*. What value did you get for *x*?
     3. Now substitute this value for *x* in Equation 1 and solve for *y*. What is your ordered pair solution for the system?

Use your findings to answer the following in sentence form:

* 1. Is the ordered pair solution the same for either variable that is eliminated? Justify your answer.
  2. Would you need to eliminate both variables to solve the problem? Justify your answer.
  3. What are some things you should consider when deciding which variable to eliminate? Is there a wrong variable to eliminate?
  4. How do you decide what constant to multiply by in order to make the chosen variable eliminate?

Use the elimination method to solve the following systems of equations. Verify your solution by substituting it into the original system.

1. 3*x* + 2*y* = 6 3. –6*x* + 5*y* = 4 4. 5*x* + 6*y* = -16

–6*x* – 3*y* = -6 7*x* – 10*y* = –8 2*x* + 10*y* = 5

## Summer Job

**Mathematical Goals**

* Model and write an inequality in two variables and solve a problem in context.
* Create two-variable linear equations and inequalities from contextual situations.
* Solve word problems involving inequalities.
* Represent constraints with inequalities.

**Common Core State Standards**

**MCC9‐12.A.REI.12** Graph the solutions to a linear inequality in two variables as a half‐plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half‐planes.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

**Introduction**

In this task, students will write a model for an inequality from the context of a word problem using real life situations. The students will then graph the inequality in two variables and analyze the solution. Students will reason quantitatively and use units to solve problems.

**Materials**

* Graph paper
* Notebook paper
* Ruler
* Pencil
* Colored Pencils

In order to raise money, you are planning to work during the summer babysitting and cleaning houses. You earn $10 per hour while babysitting and $20 per hour while cleaning houses. You need to earn at least $1000 during the summer.

1. Write an expression to represent the amount of money earned while babysitting. Be sure to choose a variable to represent the number of hours spent babysitting.

***Let b represent the number of hours spent babysitting***

***$10b represents the amount of money earned while babysitting***

1. Write an expression to represent the amount of money earned while cleaning houses.

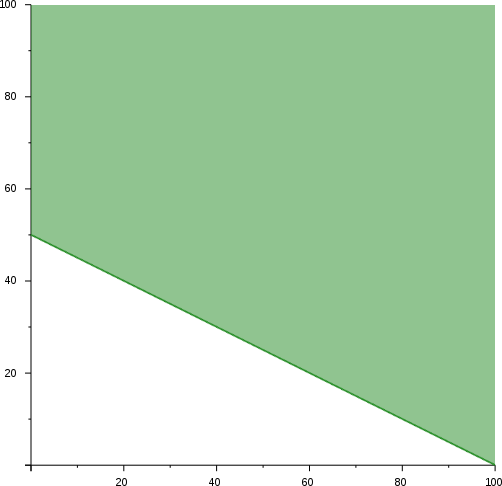
***Let c represent the number of hours spent cleaning***

***$20c represents the amount of money earned while cleaning houses.***

1. Write a mathematical model (inequality) representing the total amount of money earned over the summer from babysitting and cleaning houses.

***$10b + $20c > $1000***

1. Graph the mathematical model. Graph the hours babysitting on the *x*-axis and the hours cleaning houses on the *y*-axis.



1. Use the graph to answer the following:
   1. Why does the graph only fall in the 1st Quadrant?

***Neither the hours spent babysitting nor the hours cleaning houses can be negative.***

* 1. Is it acceptable to earn exactly $1000? What are some possible combinations of outcomes that equal exactly $1000? Where do all of the outcomes that total $1000 lie on the graph?

***Yes, it is possible to earn exactly $1000. Some possibilities include (100, 0), (20, 40), and (80, 10), but answers will vary. All of the outcomes totaling exactly $1000 lie on the line.***

* 1. Is it acceptable to earn more than $1000? What are some possible combinations of outcomes that total more than $1000? Where do all of these outcomes fall on the graph?

***Yes, it is acceptable to earn more than $1000. Some possibilities are (10, 60), and (70, 70), but answers will vary. All of the outcomes totaling more than $1000 are above the line.***

* 1. Is it acceptable to work 10 hours babysitting and 10 hours cleaning houses? Why or why not? Where does the combination of 10 hours babysitting and 10 hours cleaning houses fall on the graph? Are combinations that fall in this area a solution to the mathematical model? Why or why not?

***No it is not acceptable to work 10 hours babysitting and 10 hours cleaning houses. This combination would result in earnings of only $300 for the summer (10\*10 + 20\*10). Since you needed $1000 this is not acceptable. This combination falls below the line. Any combination that falls in the area below the line is not a solution because it would result in earnings less than $1000.***

1. How would the model change if you could only earn more than $1000? Write a new model to represent needing to earn more than $1000. How would this change the graph of the model? Would the line still be part of the solution? How would you change the line to show this? Graph the new model.

***New model: $10x + $20y > $1000***

***The line on the graph would no longer be part of the solution, therefore it would be broken and not solid.***

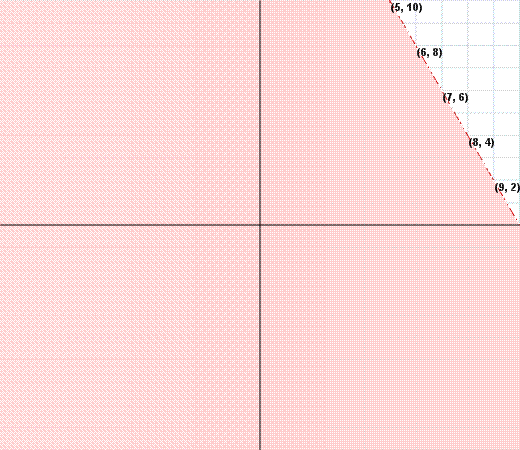
You plan to use part of the money you earned from your summer job to buy jeans and shirts for school. Jeans cost $40 per pair and shirts are $20 each. You want to spend less than $400 of your money on these items.

1. Write a mathematical model representing the amount of money spent on jeans and shirts.

***$40j + $20s < $400***

1. Graph the mathematical model. Graph the number of jeans on the *x*-axis and shirts on the *y*-axis.

|  |
| --- |
|  |



* 1. Why does the graph only fall in the 1st Quadrant?

***Neither he number of pairs of jeans nor the number of shirts purchased can be negative.***

* 1. Is it acceptable to spend less than $400? What are some possible combinations of outcomes that total less than $400? Where do all of these outcomes fall on the graph?

***It is acceptable to spend less than $400. All of the possible combinations totaling less than $400 fall below the line.***

* 1. Is it acceptable to spend exactly $400? How does the graph show this?

***It is not acceptable to spend exactly $400, therefore the line is broken.***

* 1. Is it acceptable to spend more than $400? Where do all of the combinations that total more than $400 fall on the graph?

***It is not acceptable to spend more than $400. All of the combinations totaling more than $400 are above the line on the graph.***

Summarize your knowledge of graphing inequalities in two variables by answering the following questions in sentence form:

***Answers to these questions will vary, but should demonstrate student understanding of the reasoning behind graphing inequalities.***

1. Explain the difference between a solid line and a broken line when graphing inequalities. How can you determine from the model whether the line will be solid or broken? How can you look at the graph and know if the line is part of the solution?

***Answers will vary, but a solid line indicates these combinations are part of the solution and the inequality contains an equal sign. A broken line indicates the line is not part of the solution and the inequality does not contain an equal sign.***

1. How do you determine which area of the graph of an inequality to shade? What is special about the shaded area of an inequality? What is special about the area that is not shaded?

***The area that contains solutions to the inequality is shaded. The area that is not shaded does not contain solutions to the inequality.***

## Summer Job

**Mathematical Goals**

* Model and write an inequality in two variables and solve a problem in context.
* Create two-variable linear equations and inequalities from contextual situations.
* Solve word problems involving inequalities.
* Represent constraints with inequalities.

**Common Core State Standards**

**MCC9‐12.A.REI.12** Graph the solutions to a linear inequality in two variables as a half‐plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half‐planes.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

In order to raise money, you are planning to work during the summer babysitting and cleaning houses. You earn $10 per hour while babysitting and $20 per hour while cleaning houses. You need to earn at least $1000 during the summer.

1. Write an expression to represent the amount of money earned while babysitting. Be sure to choose a variable to represent the number of hours spent babysitting.
2. Write an expression to represent the amount of money earned while cleaning houses.
3. Write a mathematical model (inequality) representing the total amount of money earned over the summer from babysitting and cleaning houses.
4. Graph the mathematical model. Graph the hours babysitting on the *x*-axis and the hours cleaning houses on the *y*-axis.



1. Use the graph to answer the following:
   1. Why does the graph only fall in the 1st Quadrant?
   2. Is it acceptable to earn exactly $1000? What are some possible combinations of outcomes that equal exactly $1000? Where do all of the outcomes that total $1000 lie on the graph?
   3. Is it acceptable to earn more than $1000? What are some possible combinations of outcomes that total more than $1000? Where do all of these outcomes fall on the graph?
   4. Is it acceptable to work 10 hours babysitting and 10 hours cleaning houses? Why or why not? Where does the combination of 10 hours babysitting and 10 hours cleaning houses fall on the graph? Are combinations that fall in this area a solution to the mathematical model? Why or why not?
2. How would the model change if you could only earn more than $1000? Write a new model to represent needing to earn more than $1000. How would this change the graph of the model? Would the line still be part of the solution? How would you change the line to show this? Graph the new model.



You plan to use part of the money you earned from your summer job to buy jeans and shirts for school. Jeans cost $40 per pair and shirts are $20 each. You want to spend less than $400 of your money on these items.

1. Write a mathematical model representing the amount of money spent on jeans and shirts.
2. Graph the mathematical model. Graph the number of jeans on the *x*-axis and shirts on the *y*-axis.



* 1. Why does the graph only fall in the 1st Quadrant?
  2. Is it acceptable to spend less than $400? What are some possible combinations of outcomes that total less than $400? Where do all of these outcomes fall on the graph?
  3. Is it acceptable to spend exactly $400? How does the graph show this?
  4. Is it acceptable to spend more than $400? Where do all of the combinations that total more than $400 fall on the graph?

Summarize your knowledge of graphing inequalities in two variables by answering the following questions in sentence form:

1. Explain the difference between a solid line and a broken line when graphing inequalities. How can you determine from the model whether the line will be solid or broken? How can you look at the graph and know if the line is part of the solution?
2. How do you determine which area of the graph of an inequality to shade? What is special about the shaded area of an inequality? What is special about the area that is not shaded?

## Graphing Inequalities

**Mathematical Goals**

* Model and write an inequality in two variables and solve a problem in context.
* Create two-variable linear equations and inequalities from contextual situations.
* Solve word problems involving inequalities.
* Represent constraints with inequalities.

**Common Core State Standards**

**MCC9‐12.A.REI.12** Graph the solutions to a linear inequality in two variables as a half‐plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half‐planes.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

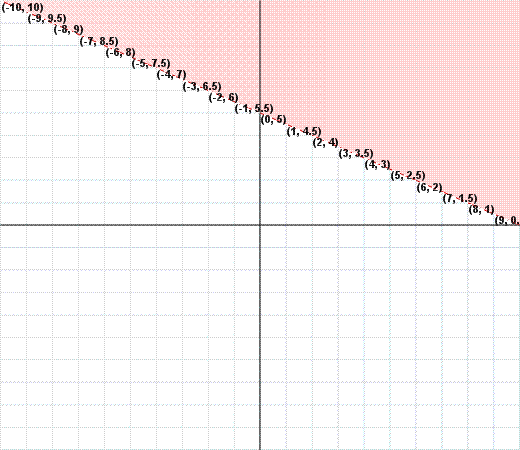
**Introduction**

In this task, students will graph two separate inequalities in two variables and analyze the graph for solutions to each. The students will then graph the two inequalities in two variables on the same coordinate system to show that the solution to both inequalities is the area where the graphs intersect.

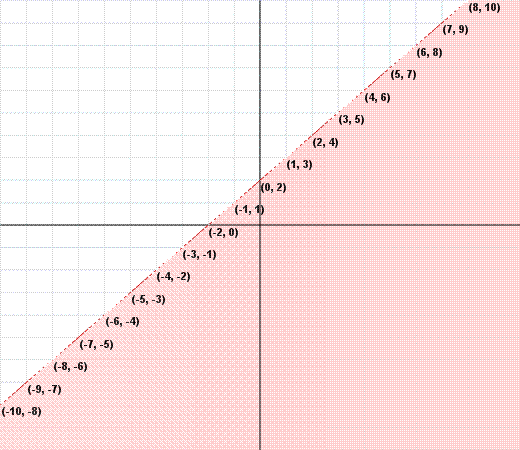
**Materials**

* Colored pencils
* Pencil
* Calculator
* Paper
* Ruler

1. Graph the inequality *y* > -½ *x* + 5. What are some solutions to the inequality?



1. Graph the inequality *y* < *x* + 2. What are some solutions to the inequality?



1. Look at both graphs.

***The main purpose of this exercise is to allow students to discover visually and conceptually where the solutions to the inequalities lie on the graph.***

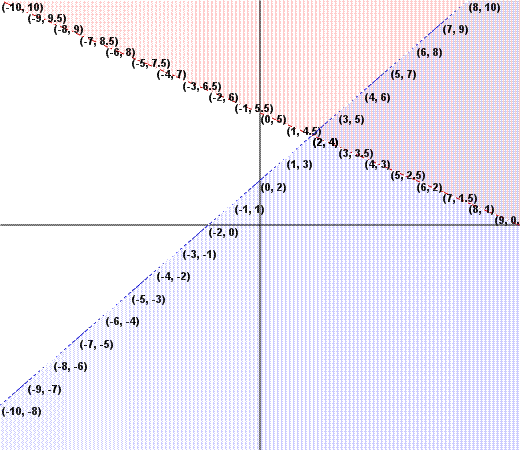
* 1. Are there any solutions that work for both inequalities? Give 3 examples.

***There are many solutions that work for both, including: (-2, 7), (4, 4), (7, 3)***

* 1. Are there any solutions that work for 1 inequality but not the other? Give 3 examples and show which inequality it works for.

***There are many solutions that work for one inequality but not the other.***

1. Graph both inequalities on the same coordinate system, using a different color to shade each.



* 1. Look at the region that is shaded in both colors. What does this region represent?

***The region shaded in both colors represents the solutions to the system.***

* 1. Look at the regions that are shaded in only 1 color. What do these regions represent?

***The regions shaded in one color represent solutions that work for one inequality, but not the other.***

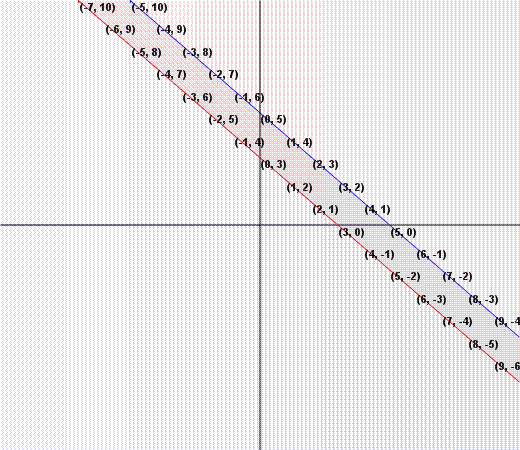
* 1. Look at the region that is not shaded. What does this region represent?

***The region that is not shaded represents combinations that are not solutions to either inequality.***

1. Graph the following system on the same coordinate grid. Use different colors for each.

*x* + *y* ≥ 3

*y* ≤ -*x* + 5



1. Give 3 coordinates that are solutions to the system.

***Answers may vary.***

1. Give 3 coordinates that are not solutions to the system.

***Answers may vary.***

1. Is a coordinate on either line a solution?

***Yes, coordinates on the line are solutions to the system.***

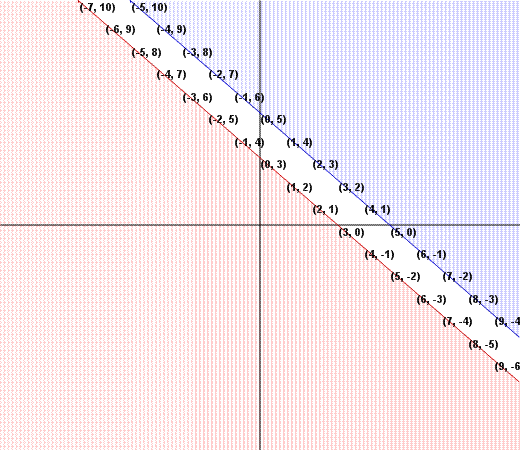
1. How would you change the inequality *x* + *y* ≥ 3 so that it would shade below the line?

***If you change the > to < , the graph will shade below.***

1. How would you change the inequality *y* ≤ -*x* + 5 so that it would shade above the line?

***If you change < to > , the graph will shade above the line.***

1. Graph the new equations from ‘d’ and ‘e’ above on the same coordinate grid. Use blue for one graph and red for the other.



* 1. What do the coordinates in blue represent?

***Each color represents solutions to one inequality, but not the other.***

* 1. What do the coordinates in red represent?

***See above.***

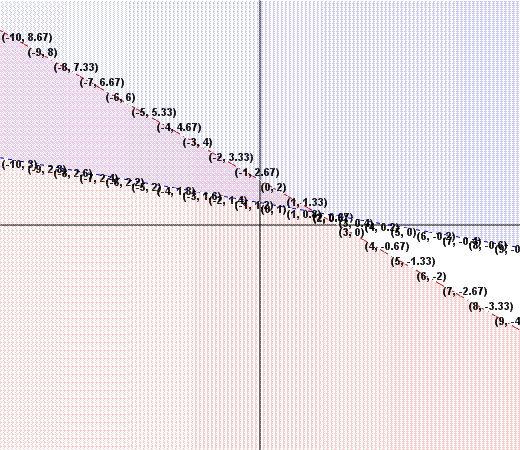
* 1. Why do the colors not overlap this time?

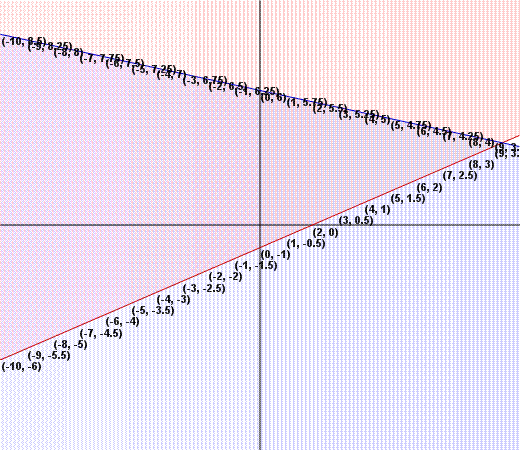
***There is no coordinate that is a solution to both inequalities. Therefore, the system has no solution.***

Graph the following on the same coordinate grid and give 3 solutions for each.

1. 2*x* + 3*y* < 6

*x* + 5*y* > 5

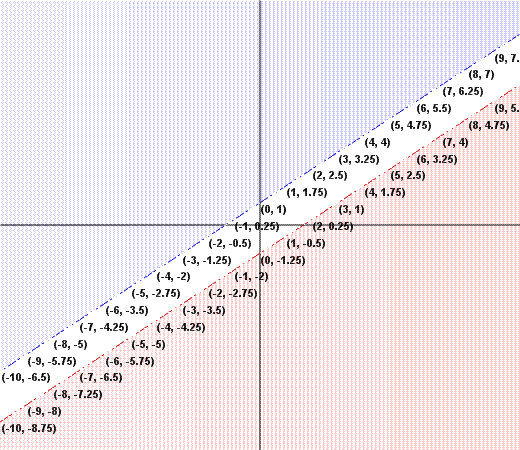


1.  *y* ≥ ½ *x* – 1

*y* ≤ -¼ *x* + 6

1. 3*x* – 4*y* > 5

*y* > ¾ *x* + 1



## Graphing Inequalities

**Mathematical Goals**

* Model and write an inequality in two variables and solve a problem in context.
* Create two-variable linear equations and inequalities from contextual situations.
* Solve word problems involving inequalities.
* Represent constraints with inequalities.

**Common Core State Standards**

**MCC9‐12.A.REI.12** Graph the solutions to a linear inequality in two variables as a half‐plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half‐planes.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

1. Graph the inequality *y* > – ½ *x* + 5. What are some solutions to the inequality?



1. Graph the inequality *y* < *x* + 2. What are some solutions to the inequality?



1. Look at both graphs.
   1. Are there any solutions that work for both inequalities? Give 3 examples.
   2. Are there any solutions that work for 1 inequality but not the other? Give 3 examples and show which inequality it works for.
2. Graph both inequalities on the same coordinate system, using a different color to shade each.



* 1. Look at the region that is shaded in both colors. What does this region represent?
  2. Look at the regions that are shaded in only 1 color. What do these regions represent?
  3. Look at the region that is not shaded. What does this region represent?

1. Graph the following system on the same coordinate grid. Use different colors for each.

*x* + *y* ≥ 3

*y* ≤ –*x* + 5



1. Give 3 coordinates that are solutions to the system.
2. Give 3 coordinates that are not solutions to the system.
3. Is a coordinate on either line a solution?
4. How would you change the inequality *x* + *y* ≥ 3 so that it would shade below the line?
5. How would you change the inequality *y* ≤ – *x* + 5 so that it would shade above the line?
6. Graph the new equations from ‘d’ and ‘e’ above on the same coordinate grid. Use blue for one graph and red for the other.



* 1. What do the coordinates in blue represent?
  2. What do the coordinates in red represent?
  3. Why do the colors not overlap this time?

Graph the following on the same coordinate grid and give 3 solutions for each.

1. 2*x* + 3*y* < 6

*x* + 5*y* > 5



1. *y* ≥ ½ *x* – 1

*y* ≤ –¼ *x* + 6

1. 3*x* – 4*y* > 5

*y* > ¾ *x* + 1

## Culminating Task: Family Outing

**Mathematical Goals**

* Model and write an inequality in two variables and solve a problem in context.
* Create two-variable linear equations and inequalities from contextual situations.
* Solve word problems involving inequalities.
* Represent constraints with inequalities.

**Common Core State Standards**

**MCC9‐12.A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**MCC9‐12.A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

**MCC9‐12.A.REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

**MCC9‐12.A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**MCC9‐12.A.REI.12** Graph the solutions to a linear inequality in two variables as a half‐plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half‐planes.

**MCC9‐12.A.REI.12** Graph the solutions to a linear inequality in two variables as a half‐plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half‐planes.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

**Introduction**

In this task, students will write a model for an inequality from the context of a word problem using real life situations. The students will then graph the inequality in two variables and analyze the solution. Students will reason quantitatively and use units to solve problems.

**Materials**

* Pencil
* Colored Pencils
* Ruler
* Calculator

You and your family are planning to rent a van for a 1 day trip to Family Fun Amusement Park in Friendly Town. For the van your family wants, the Wheels and Deals Car Rental Agency charges $25 per day plus 50 cents per mile to rent the van. The Cars R Us Rental Agency charges $40 per day plus 25 cents per mile to rent the same type van.

1. Write a mathematical model to represent the cost of renting a van from the Wheels and Deals Agency for 1 day.

***C = $25 + $.50m***

* 1. Do the units matter for this equation?

***Yes, the units matter. Both the cost per day and the cost per mile should be in the same unit.***

* 1. Use the equation to determine the cost for renting the van from this agency for 1 day and driving 40 miles.

***C = $25(1) + $.50(40)***

***C = $25 + $20***

***C = $45***

1. Write a mathematical model to represent the cost of renting from the Cars R Us Agency for 1 day.

***C = $40 + $.25m***

* 1. Do the units for this equation match the units for the equation in problem 1? Does this matter when comparing the 2 equations?

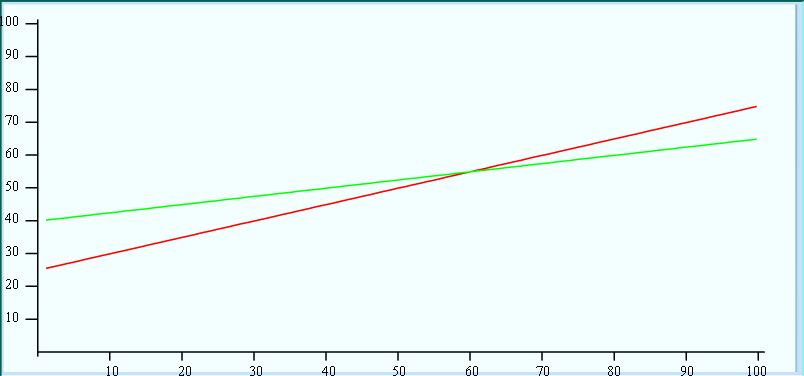
***The units should be the same for both equations.***

* 1. Use the equation from ‘2a’ to determine the cost for renting the van from Cars R Us for 1 day and driving 40 miles.

***C = $40(1) + $.25(40)***

***C = $50***

1. Graph the 2 models on the same coordinate system. Be sure to extend the lines until they intersect.



* 1. Where do the 2 lines intersect?

***(60, 55) After 60 miles, the cost for the rental will be $55.***

* 1. What does the point of intersection represent?

***The point represents the number of miles for which the cost of the rental will be the same for both agencies.***

* 1. When is it cheaper to rent from Wheels and Deals?

***It is cheaper to rent from Wheels and Deals when you are driving less than 60 miles.***

* 1. When is it cheaper to rent from Cars R Us?

***It is cheaper to rent from Cars R Us when you are driving more than 60 miles.***

1. Friendly Town is approximately 80 miles from your home town. Which agency should you choose? Justify your answer.

***You should choose the Cars R Us agency because the cost of renting from them would be approximately $60. The cost for renting from Wheels and Deals would be approximately $65.***

When you leave the car rental agency, your father goes to the Fill ‘er Up Convenience Store for gas. The gas hand indicates the van is on empty, so your father plans to fill the tank. Gas at the station is $3.49 per gallon.

1. If your father spends $78 on gas, approximately how many gallons did he purchase?

***$78 = $3.49\*g***

***g = $78/$3.49***

***He purchased approximately 22 gallons of gas.***

While in the store, your father purchases drinks for the six people in your van. Part of your family wants coffee and the rest want a soda.

1. Coffee in the store costs $.49 per cup and sodas are $1.29 each. The cost of the drinks before tax was $6.14.
   1. Write a mathematical model that represents the total number of cups of coffee and sodas.

***c + s = 6***

* 1. Write a mathematical model that represents the cost of the coffee and soda.

***$.49c + $1.29s = $6.14***

* 1. Solve the system of equations using the elimination method.

***-.49(c + s) = –.49(6)***

***–..49c –. .49s = -2.94***

***+ .49c + 1.29s = 6.14***

***.8s = 3.2***

***s = 4***

***c + 4 = 6***

***c = 2***

***Your father purchased 2 cups of coffee and 4 sodas.***

When you arrive in Friendly Town at the Family Fun Amusement Park, the 6 people in your family pair up to enter the park. You and your brother decide to enter and ride together. The cost to enter the park is $10, with each ride costing $2.

1. You bring $55 to the park. You must pay to enter the park and you budget an additional $10 for food. Write and solve an inequality to determine the maximum number of rides you can ride. Explain your answer.

***$10 + $10 + $2r < $55***

***$2r < $35***

***r < 17.5***

***The maximum number rides you can ride is 17, because you can’t ride half of a ride.***

1. Your brother brings $70 to the park and budgets $12 for food. How many more rides can he ride than you? Explain your answer.

***$10 + $12 + 2r < $70***

***r < 24 rides***

***Your brother can ride up to 24 rides. You can ride up to 17. Therefore, he can ride 7 more rides than you.***

Inside the park, there are 2 vendors that sell popcorn and cotton candy. Jiffy Snacks sells both for $2.50 per bag. Quick Eats has cotton candy for $4 per bag and popcorn for $2 per bag.

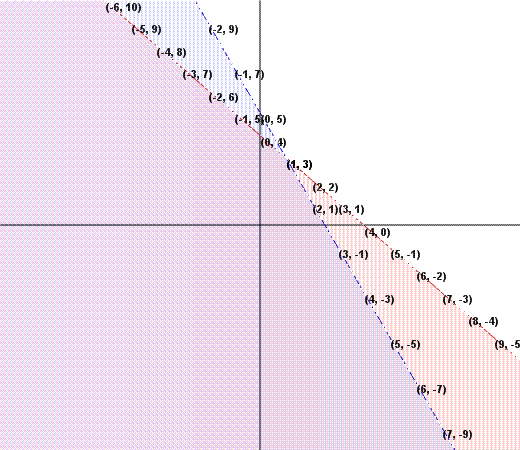
1. If you use the $10 you budgeted for food, write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Jiffy Snacks.

***$2.50c + $2.50p < $10***

1. Write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Quick Eats.

***$4.00c + $2.00p < $10***

1. Graph the system of inequalities. Give two combinations that work for both vendors.



1. Assuming you purchase at least one of each, what is the maximum number of bags of cotton candy and popcorn that work for both equations?

***The maximum that works for both equations is 1 bag of cotton candy and 3 bags of popcorn.***

When you leave the park, your father notices that you have used ¾ of the tank of gas you purchased before you left.

1. Do you have enough gas to get home? Justify your answer.

***The methods for answering this question may vary, but you do not have enough gas to get home. You have used approximately 17 of the 22 gallons you purchased earlier.***

***You will need approximately 12 gallons of gas to get home.***

1. Your father wants to purchase enough gas to get home, but not leave extra in the tank when the van is returned to the rental agency. Approximately how many more gallons should he purchase? Justify your answer.

***See above.***

**Family Outing**

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   1. Do the units matter for this equation?
   2. Use the equation to determine the cost for renting the van from this agency for 1 day and driving 40 miles.
2. Write a mathematical model to represent the cost of renting from the Cars R Us Agency for 1 day.
   1. Do the units for this equation match the units for the equation in problem 1? Does this matter when comparing the 2 equations?
   2. Use the equation from ‘2a’ to determine the cost for renting the van from Cars R Us for 1 day and driving 40 miles.
3. Graph the 2 models on the same coordinate system. Be sure to extend the lines until they intersect. 
   1. Where do the 2 lines intersect?
   2. What does the point of intersection represent?
   3. When is it cheaper to rent from Wheels and Deals?
   4. When is it cheaper to rent from Cars R Us?
4. Friendly Town is approximately 80 miles from your home town. Which agency should you choose? Justify your answer.

When you leave the car rental agency, your father goes to the Fill ‘er Up Convenience Store for gas. The gas hand indicates the van is on empty, so your father plans to fill the tank. Gas at the station is $3.49 per gallon.

1. If your father spends $78 on gas, approximately how many gallons did he purchase?

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   1. Write a mathematical model that represents the total number of cups of coffee and sodas.
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2. Your brother brings $70 to the park and budgets $12 for food. How many more rides can he ride than you? Explain your answer.

Inside the park, there are 2 vendors that sell popcorn and cotton candy. Jiffy Snacks sells both for $2.50 per bag. Quick Eats has cotton candy for $4 per bag and popcorn for $2 per bag.

1. If you use the $10 you budgeted for food, write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Jiffy Snacks.
2. Write an inequality to model the possible combinations of popcorn and cotton candy you can purchase from Quick Eats.
3. Graph the system of inequalities. Give two combinations that work for both vendors.



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1. Do you have enough gas to get home? Justify your answer.
2. Your father wants to purchase enough gas to get home, but not leave extra in the tank when the van is returned to the rental agency. Approximately how many more gallons should he purchase? Justify your answer.